THE SMALL GRAVITATIONAL TORQUE FORCED A ROTATING TRIAXIAL BODY

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Abstract

In his paper I examine influence of small gravitational torque on rotation of elongated triaxial bodies. The Hamiltonian of a body moving in central gravitational field separates on two parts: orbital movement about central body and a rotation around the body mass center. For the small bodies like asteroids the separation spin-orbit constant has rate 10-12 of total energy and orbital and rotational motion are almost independent. This way we may consider orbital motion as a known function of time or true anomaly. Using the Hamiltonian I found gravitational torque affecting triaxial body in quadruple approximation. The Euler-Liouville equation is a system of non-linear differential equations. Position of the body is described by six variables: vector $\vec{R}$ in inertial reference system and three Euler angle: φ, ψ and ϑ rigidly bounded to the principal axes of the body inertia tensor. The rotational motion is described by angular velocity (vector $\vec{\omega}$) or angular momentum vector $\overrightarrow{L}=\hat{I}\vec{\omega}$ or $\hat{I} = diag\left(I_x, I_y, I_z\right)$ or $\hat{I} = I_z diag(a; b; 1)$ denotes diagonal inertia tensor of the body) and three Euler angle. A numerical resolution of gravitationally disturbed Euler- Liouville equation is compared with the undisturbed one. This solution is well known as the Poinset solution of the free body rotation. Modelling of rotational motion is a great interest because its connections to astronomical measurements of asteroids physical properties. I found that direction of spin-vector of a rotating body in NPA state of motion changes markedly when forced by gravitational torque.

Key words: gravitational torque, asteroids rotation, tumbling asteroids, numerical computation

1 Introduction

A rotational motion of asteroids is still interesting physical problem, because of its relationships with the development and evolution of the Solar System. There are two fundamental problems in dynamics of rotational motion. One, what is the source of observed rotational motion, it means, what kind of interaction are responsible for dynamics of asteroids rotation. Second, do these interactions disturb observed rotational state of small celestial bodies sufficiently in realistic time scale (dozens years), to bring measurable changes.

There are known following dynamical agents: tidal forces, random collisions, YORP effect (related to infrared radiation of asteroid surface) and gravitational torque of the Sun and planets [2,3,7,17,18,21]. As main long-term factors are regarded tidal forces and YORP effect. The first accounts for dissipative loss of kinetic energy, the second and third may change rotational state of a body [2,3,7,12,17,18,20,21]. They are beyond of interest in this paper.

The physical variables describing rotational motion are angular velocity $\vec{\omega}$, period of rotation T_a (if a motion is periodic), angular momentum $\vec{L}$, inertia tensor of a body and its kinetic energy T. An important measurable parameter is also an angular momentum of obliquity against orbital plane of the asteroid [3,13]. Today we know these parameters for about thousand asteroids [10,22]. The angular momentum direction (an asteroid pole) describes two angles: latitude β and longitude λ (see fig.1).

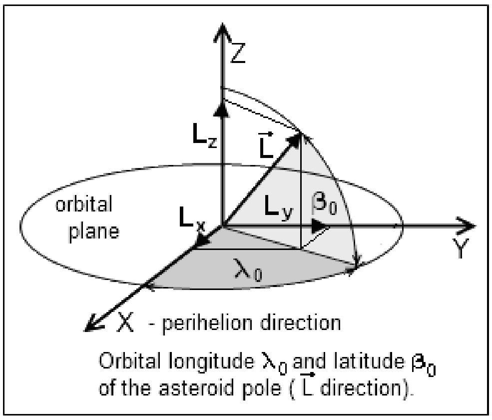

Figure 1. The asteroid pole coordinates

These data are released to orbital or ecliptic plane. None asteroids with β=0 exist; most of them have β nearly $\pm 90^o$. The distribution shape of longitude is nearly uniform [10,13].

In absence of external torque the angular momentum is constant of the motion:

$$L^2 = I_x^2\omega_x^2 + I_y^2\omega_y^2 + I_z^2\omega_z^2 = \text{const}.$$

Then possible kinetic energy is depending on angular velocity direction and for constant L it varies from $T_{min} = L^2 / I_z$ to $T_{max} = L^2 / I_x$

($I = diag(I_x, I_y, I_z); I_x \le I_y \le I_z$ -inertia tensor of a body).

Important parameter is so called dynamic inertia I_D[3,11]:

$$I_D = \frac{L^2}{2T} \qquad I_x \le I_D \le I_z .$$

If the rotation axis is x or z (principal axes), $I_D = I_x$ or $I_D = I_z$ and we say, that the body is in Long Axis Mode (LAM) or in Short Axis Mode (SAM) [3,11]. We distinguish SAM+ and SAM- if body rotation is prograde or retrograde versus orbital motion and respectively distinguish LAM+ and LAM-.

I_D is constant in the unperturbed motion. Note that in SAM motion the kinetic energy is minimal, so this is equilibrium state and in LAM motion the kinetic energy is maximal, so this is non-steady state. If $I_D = I_y$ equilibrium is unstable the rotational motion becomes chaotic. Intermediate I_D means unstable motion known as tumbling. These asteroids are called NPA (non-principal axis) rotators [13]. Long term interactions (tidal forces) lead to SAM motion and we observe that the most known asteroids are in SAM state [6,10]. The angular velocity of asteroids shows obvious dependency on asteroids diameter. According to data from [6,13] medium period of rotational motion for most asteroids is near $T_a \approx 6hr$, but for small asteroids ($D \approx 1km$) it is about 5hr and for greater ($D \approx 100km$) $T_a \approx 8 \div 10hr$. This means $2.5 \div 3rev/day$. Since 1980 a dozens asteroids were discovered with significantly greater periods of rotation range of $40 \div 1200hr$ - very slow rotated asteroids [5]. The kinetic energy of these asteroids is considerably less and gravitational torque is strong enough to disturb free rotational motion in reasonable time. There is possibility to measure deviation from undisturbed Euler-Liouville equation, especially for spin pole coordinates. A problem of gravitational torque interaction was discussed in some papers [10,12,15,16,20], generally for trajectories of spin pole on plane $\beta\lambda$. Usual assumption SAM state and unforced rotational motion is made, so problem becomes two-dimensional in obliquity and in value of an angular momentum. In most papers discussion of rotational motion of asteroids is based on the well known Poinset solution of unperturbed Euler-Liouville equations [14]. In article [15] Ryabova do the same, but additionally calculate differences between perturbed and unperturbed solutions of Euler-Liouville equations. She concluded, that the difference is in range of the measurements errors, so it is no reason to perturbed calculation. It is true, but medium period of asteroid 1620 Geographos rotation was estimated be 5.2hr and it is too small to obtain marked difference (very fast rotation).

2 Gravitationally disturbed rotational motion of asteroids.

The main problems of this work is, if gravitational torque changes significantly unperturbed solutions of Euler-Liouville equations and if this changes develops in reasonable time (no more than dozens years). In this work we study small gravitational torque and its effects on rotational motion of elongated body of irregular shape. To write equations describing this motion, we have to define some reference systems. The first is the orbital inertial system XYZ with Z axis perpendicular to orbit plane and X axis parallel to aphelion line. The second system xyz is fixed to principal axis of inertia tensor $\hat{I} = \mathrm{diag}(I_x, I_y, I_z)$ of the body (BFCM shortly) with origin in the body mass centre; x is relative to the minimum inertia moment I_x, y to the intermediate moment I_y and z to the maximum moment I_z. The systems are related to other by Euler's angle and rotation matrix from an inertial system to the body fixed system is $\hat{A}(\varphi, \psi, \vartheta)$:

$$\hat{A}(\varphi,\psi,\vartheta) = \begin{vmatrix} \cos\varphi\cos\psi - \sin\varphi\sin\psi\cos\vartheta & \sin\varphi\cos\psi + \cos\varphi\sin\psi\cos\vartheta & \sin\psi\sin\vartheta \\ -\cos\varphi\sin\psi - \sin\varphi\cos\psi\cos\vartheta & -\sin\varphi\sin\psi + \cos\varphi\cos\psi\cos\vartheta & \cos\psi\sin\vartheta \\ \sin\varphi\sin\vartheta & -\cos\varphi\sin\vartheta & \cos\vartheta \end{vmatrix}$$

(1)

The Hamiltonian describing the motion of an asteroid has form:

$$H = \frac{p_R^2}{2m} + \frac{L_o^2}{2mR^2} + \frac{1}{2}\vec{\omega}\hat{I}\vec{\omega} + U(R,\varphi,\psi,\vartheta) \qquad (2)$$

where $\overrightarrow{p_R} = m\dot{\vec{R}}$ is asteroid orbital momentum, $\vec{L}_o = \vec{R} \times \vec{p}_R$ - orbital angular momentum, $T = \frac{1}{2}\vec{\omega}\hat{I}\vec{\omega}$ - kinetic energy of rotational motion and $U(R,\varphi,\psi,\vartheta)$ - the potential energy of the body in a gravitational field. If we disregard solar oblateness and potential non-centricity, we may calculate $U(R,\varphi,\psi,\vartheta)$ as an integral:

$$U(R,\varphi,\psi,\vartheta) = \int_V \frac{GM_S\rho(\vec{r})dV}{|\vec{R} - \vec{r}|} \qquad (3)$$

where integrating is made over whole volume of an asteroid.

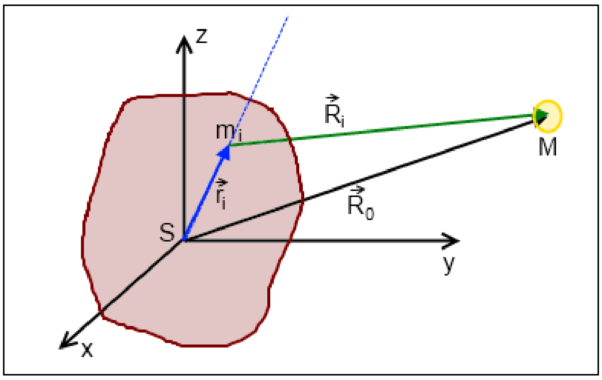

Figure 2. The body fixed reference frame: S – center of the mass.

If we expand potential into Taylor series around vector $\vec{R}_0$ denoting center mass of the body, we may write the Hamiltonian as two parts in quadruple approximation:

$$H = H_0 + H_1 = \frac{p_R^2}{2m} + \frac{L_o^2}{2mR^2} - \frac{GMm}{R} + \frac{1}{2}\vec{\omega}\hat{I}\vec{\omega} - \frac{GM}{R^3}(\mathrm{Tr}\hat{I} - \vec{h}\hat{I}\vec{h})$$

.

$$H_0 = \frac{p_R^2}{2m} + \frac{L_o^2}{2mR^2} - \frac{GMm}{R};$$
$$H_1 = \frac{1}{2}\vec{\omega}\hat{I}\vec{\omega} - \frac{GM}{R^3}(\mathrm{Tr}\hat{I} - \vec{h}\hat{I}\vec{h}). \qquad (4)$$

A vector $\vec{h}$ is a unit vector in $\vec{R}_0$ direction: $\vec{h} = \frac{\vec{R}_0}{R}$. A spin-orbit coupling constant $D \cong \frac{GMI_z}{R^3} : \frac{GMm}{R} = \frac{I_z}{mR^2}$ for almost all known asteroids is less than $\frac{d^2}{A.U.^2}$, where d is a medium asteroid diameter and A.U. is astronomical unit. If d is in range of a few km, then $D<10^{-14}$. So the orbital motion is independent from rotational one and $\vec{R}_0$ is regarded as a known function of the time. We may introduce a true anomaly u instead the time. Then $R(u) = \frac{A\sqrt{1-e^2}}{1+e\cos u}$; A is the great half-axis of the asteroid orbit and e is its eccentricity. External torque due to the second part of potential energy is calculated in BFCM reference system equals [1,8]:

$$\overrightarrow{M}(u,\varphi,\psi,\vartheta) = \frac{3GM}{R^3(u)}\begin{vmatrix}(I_z - I_y)h_y h_z\\(I_x - I_z)h_x h_z\\(I_y - I_x)h_x h_y\end{vmatrix} \tag{5}$$

where vector $\vec{h}$ in the same reference system is:

$$\vec{h} = \frac{\overrightarrow{R}}{R} = \begin{vmatrix}-\cos(\varphi-u)\cos\psi + \sin(\varphi-u)\sin\psi\cos\theta\\ \cos(\varphi-u)\sin\psi + \sin(\varphi-u)\cos\psi\cos\theta\\ -\sin(\varphi-u)\sin\vartheta\end{vmatrix} \tag{6}$$

Introducing true anomaly instead time in Euler-Liouville equations we must change the time derivative by differentiation with respect to u variable. Due to conservation law the angular orbital momentum is constant and (B – small half-axis):

$$\frac{1}{2}R^2(u)\dot{u} = \frac{L_0}{2\mu} = \frac{\pi AB}{T_a} = \mathrm{constant} \tag{7}$$

Hence we obtain:

$$\frac{d}{dt} = \dot{u}\frac{d}{du} = \frac{2\pi AB}{R^2(u)T_a}\frac{d}{du} = \frac{\omega_a(1+e\cos u)^2}{(1-e^2)^{\frac{3}{2}}}\frac{d}{du}; \tag{8}$$

Here T_a and ω_a are a period of orbital motion and angular circular velocity of an asteroid. It follows from third Kepler's law: $\frac{G(M+m)}{A^3} \approx \frac{GM}{A^3} = \omega_a^2$.

We introduce dimensionless variables: angular velocity $\overrightarrow{w} = \frac{\overrightarrow{\omega}}{\omega_a}$ and inertia tensor of the body:

$$\hat{J} = \frac{\hat{I}}{I_Z} = \mathrm{diag}(\frac{I_x}{I_z},\frac{I_x}{I_z},\frac{I_x}{I_z}) = \mathrm{diag}(a,b,1). \tag{9}$$

Now we can re-write Euler-Liouville equations in the new variables:

$$aw'_x = (1-b)[-\frac{(1-e^2)^{\frac{3}{2}}}{(1+e\cos u)^2}w_y w_z + 3\frac{(1+e\cos u)}{(1-e^2)^{\frac{3}{2}}}h_y h_z];$$

$$bw'_y = (1-a)[\frac{(1-e^2)^{\frac{3}{2}}}{(1+e\cos u)^2}w_x w_z - 3\frac{(1+e\cos u)}{(1-e^2)^{\frac{3}{2}}}h_x h_z]; \tag{10}$$

$$w_z' = (b - a)[-\frac{(1-e^2)^{\frac{3}{2}}}{(1+e\cos u)^2} w_x w_y + 3\frac{(1+e\cos u)}{(1-e^2)^{\frac{3}{2}}} h_x h_y];$$

Here apostrophe denotes a differentiation with respect to u variable (true anomaly). In the same variables, the Euler equation becomes a following form:

$$\varphi' = \frac{(1-e^2)^{\frac{3}{2}}}{(1+e\cos u)^2} \frac{w_x \sin\psi + w_y \cos\psi}{\sin\vartheta};$$

$$\psi' = \frac{(1-e^2)^{\frac{3}{2}}}{(1+e\cos u)^2} w_z - \varphi'\cos\vartheta; \quad (11)$$

$$\vartheta' = \frac{(1-e^2)^{\frac{3}{2}}}{(1+e\cos u)^2} (w_x \cos\psi - w_y \sin\psi).$$

The two sets of above differential equations describe a rotational motion of a rigid body under gravitational torque. Because an angular velocity in ω_a unit is in range of 20÷400 (for slowly rotating asteroid with rotation period T~60÷1200hr [5], the second terms in equations (10) is significantly less than the first one. This mean, that gravitational torque is a very small disturbance of nearly free rotational motion of the asteroids. In this paper we examine, when this small action is noticeable and how long observation time is needed to discover its influence. The observed parameters of rotational motion of the asteroids are their rotational periods and angular momentum directions. In absence of gravitational torque kinetic energy of rotation and angular momentum are a constant of the motion. For the triaxial body a part of energy conjugated to rotational motion is:

$$E = \frac{1}{2}\vec{\omega}\hat{I}\vec{\omega} - \frac{3GM}{R^3}(Tr\hat{I} - \vec{h}\hat{I}\vec{h}) \quad (12)$$

The second term is too small to perturb orbital motion, but may be significantly large to change kinetic energy or angular momentum of rotational motion. In $I_z\omega_a^2$ units we may rewrite energy as dimensionless quantity:

$$E = \frac{1}{2}(aw_x^2 + bw_y^2 + w_z^2) - \frac{1}{2}(\frac{1+e\cos u}{1-e^2})^3[1 + a + b - 3(ah_x^2 + bh_y^2 + h_z^2)] \quad (13)$$

The potential energy is far smaller than kinetic one and thus the kinetic energy is nearly constant. The changes are periodic, because periodic is potential energy. It changes periodically, because periodic are both orbital and rotational motion of asteroids.

In the inertial reference frame direction of the angular momentum is given by orbital longitude λ_0 and latitude β_0 of the asteroid pole. For our investigation it is not necessary to have knowledge about an asteroid pole in solar ecliptic or terrestrial equatorial reference frame, because we calculate only a difference between free and gravitationally disturbed rotational motion. It is the same in all references frames. If we resolve Euler-Liouville equation in BFCM frame, we get the angular momentum in an inertial orbital reference frame using reverse matrix $\hat{A}$. The main experimental problem is, if the difference achieves measurable magnitude in reasonable time interval. Sparse authors have investigated this problem [3,15,19] and some of them have concluded, that the difference is comparable to measurement error[15]. They have taken into account time of available observations from the last a few dozen years. All of them made simplified assumption the asteroids rotate about principal axis of maximum moments of inertia. Dynamics of a free rotated body describes its kinetic energy T and angular momentum. If external torques doesn't act, angular momentum vector and rotational kinetic energy are constant regardless of initial conditions and body shape.

3 The computational procedures.

In this work only a regular periodical forces are taken into account, so for numerical solving Euler-Liouville equation we may use well known standard Adams or BDF algorithms with variable time step[9]. Every calculation was performed by "ode" procedure on Scilab platform. It is very fast and effective algorithm, sufficient for long time procedures. "Ode" procedure based on Adams and BDF algorithms is absolutely stable and convergent for any time step. Some problems are fast and slow variables, because for one turn of true anomaly in some cases there is 200 and more turns some Euler angles. Most authors provides an averaging solar gravitational torque over fast variable ψ due to rotation around principal body axis [1,6]. This simplifies problem, but seems sometimes not reasonable. Long period simulation exhibits properties not evident in short period resolutions, so we solve unchanged Euler-Liouville equations. The solution is 6-dimensional time dependent discrete vector $X^T = [wx, wy, wz, \varphi, \psi, \vartheta]$. To avoid errors for long period, time step must be sufficiently small, so numerical resolution consist of several million of points and is multidimensional matrix. This resolution contains all information about remain physical parameters, like kinetic energy and vector of angular momentum. The following procedure is used. The first step is solving numerical dynamical equation of motion. Because the solution is dense in sense very large number of terms for every turn of asteroid about the Sun and graphical procedures take to many times, far greater then solving the equation, we make special procedure of rarefying. For graphical presentation enough take only a small percent of points from resolution and in this work we chose every N-th term from discrete vector X. N is any whole number. For clarity variable u (true anomaly) is back transformed to time by following formula [14]:

$$t = \frac{\mu}{L_0} \int_0^u \frac{p^2}{(1 + e\cos u)^2} du \tag{14}$$

This function is analytically integrable and is periodical function of upper limit u. In explicit form:

$$t(u) = \frac{T_a}{2\pi}\{2\pi \cdot \mathrm{int}(\frac{u+\pi}{2\pi}) + \sqrt{1-e^2}[\frac{2}{\sqrt{1-e^2}}\mathrm{arctg}(\sqrt{\frac{1-e}{1+e}}\mathrm{tg}\frac{u}{2}) - \frac{e \cdot \sin u}{1+e\cos u}]\} \tag{15}$$

By this formula results may be presented in convenient time dependant form. Next step procedure is computing kinetic energy and length of angular velocity and angular momentum as a discrete function of time. Last step needs conversion of vectors to inertial orbital reference frame to calculate angular coordinates of asteroid pole: latitude β_0 and longitude λ_0. Using Euler angles, we easy find rotation matrix $\hat{A}^{-1}(t) = \hat{A}^T(\varphi(t), \psi(t), \vartheta(t))$ by formula (1) and transform vectors $\vec{w}$ and $\vec{L}$ to orbital reference frame:

$$\vec{w}_I = \hat{A}^T \cdot \vec{w}; \vec{L}_I = \hat{A}^T \cdot \vec{L}; \tag{16}$$

From new coordinates of vector $\vec{L}_I$ and according fig.1, we find:

$$\lambda_0 = \mathrm{arctg}(\frac{L_{Iy}}{\sqrt{L_{Ix}^2 + L_{Iy}^2}}); \beta_0 = \arccos(\frac{L_{Iz}}{L}) \tag{17}.$$

The same formulas we use to vector $\vec{w}_I$. Some caution must be preserved because arctg function is ambiguous.

4 The changes of rotational motion gravitational forced asteroid. The numerical results.

The numerical computations were done systematically for every rotational mode (SAM, LAM, NPA) and chosen asteroid shapes (represented by dimensionless tensor of inertia in diagonal form). The case of asteroids with axial symmetry we excluded, because there exist some analytical solution of Euler-Liouville equation. There are two main group (inside them numerous families) of asteroids observed in the Solar System: main belt asteroids (MBA with semimajor axis A>2.0A.U.) and near Earth asteroids (NEA with semimajor axis A<2.0A.U.). According Kepler's law orbital periods of MBA asteroids T_a>2.82years and T_a<2.82years for NEA asteroids. For slow rotation for periods 40÷800hr it means in dimensionless units:

$$11 \cdot T_a < w < 220 \cdot T_a$$

where T_a (in years) is orbital period of asteroid, regardless of orbital semimajor axis and family adherence. This is very wide range of possible angular dimensionless velocity, so we have done simulation for some chosen values near upper and lower limits. Asteroids shape is represented by dimensionless tensor of inertia. If we assume medium ellipsoidal shape of asteroids and its axes sizes $A_z<A_y<A_x$, then:

$$\hat{J} = \mathrm{diag}(a, b, 1) = \mathrm{diag}\left(\frac{A_y^2 + A_z^2}{A_x^2 + A_y^2}, \frac{A_x^2 + A_z^2}{A_x^2 + A_y^2}, 1\right).$$

Reverse solution for $A_z<A_y<A_x$ exist only, if a>1-b and a<b in the darkest area on plot (fig. 3), so it means, that only b>0.5 has physical meaning.

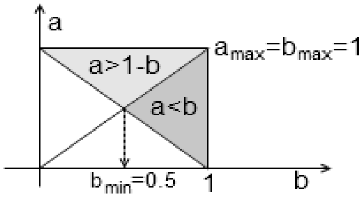

Figure 3. Possible a and b value.

From known for many asteroids data estimated by radar and telescope observation we may accept wide range of admissible a and b value [6,10,22]. So we made simulation for slight and strong elongated asteroids.

The first computations were done for SAM state (rotation along shortest axe). How we expected, changes in rotational state are negligible, regardless of angular velocity, asteroid shape and initial conditions. The angular velocity and angular momentum and kinetic energy of a rotation are a bit greater then without interaction (fig. 4,5, computed for T_a=3.2yr and wz=300).

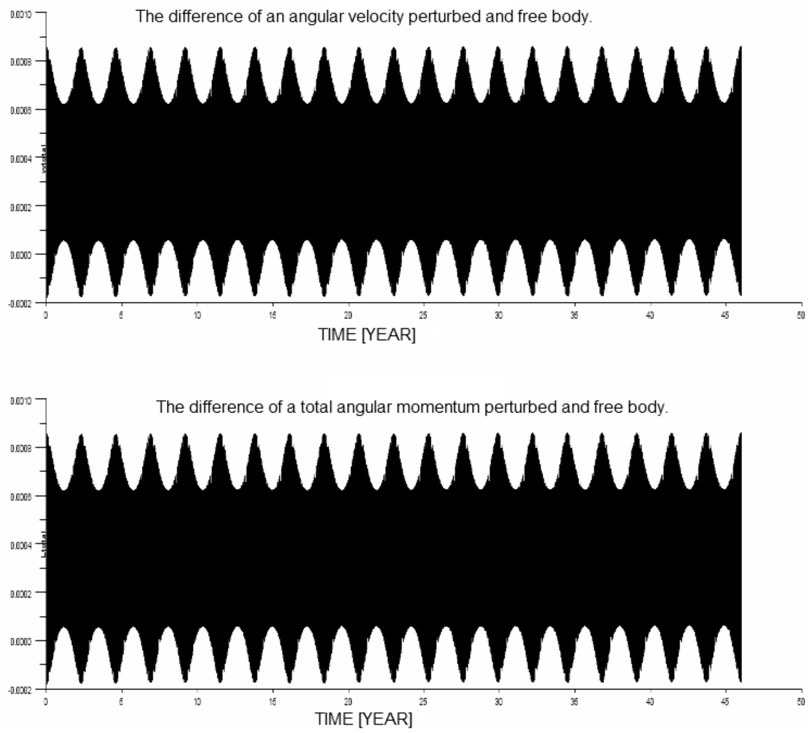

Figure 4. The small growth of total angular velocity and angular momentum gravitationally perturbed body.

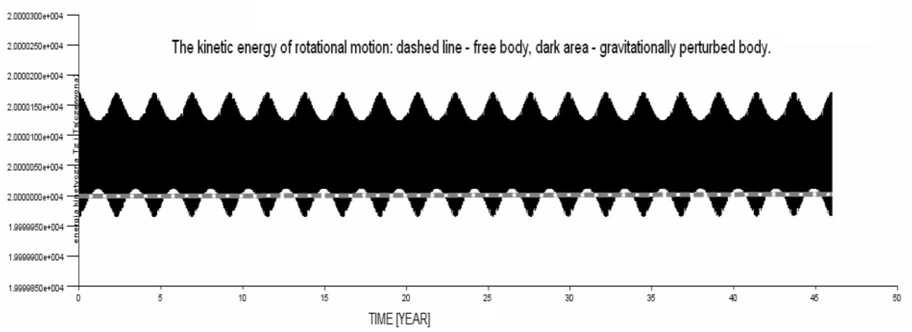

Figure 5. Kinetic energy of the rotational motion.

The rotation axe becomes precessing and nutating along initial rotation axe, but amplitude of these motions are very small (see figure 6). The same results confirm Scheeres D.J. et al. [6]. Due to the oscillating torque, all differences between free and perturbed motions are periodic and the period coincides with the period of orbital motion.

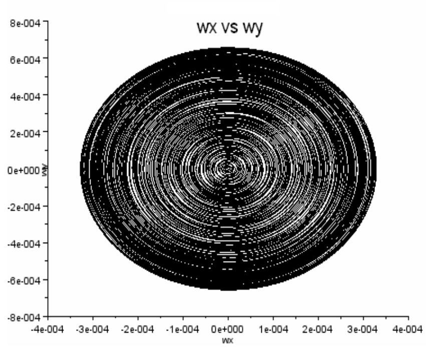

Figure 6. Precession of a z axe perturbed body near the SAM state (wx, wy – transversal components of angular velocity).

The directions of both angular velocity and angular momentum remain almost unchanged; a longitude amplitude is some greater (24' vs. 0.1') than latitude one (figure 7).

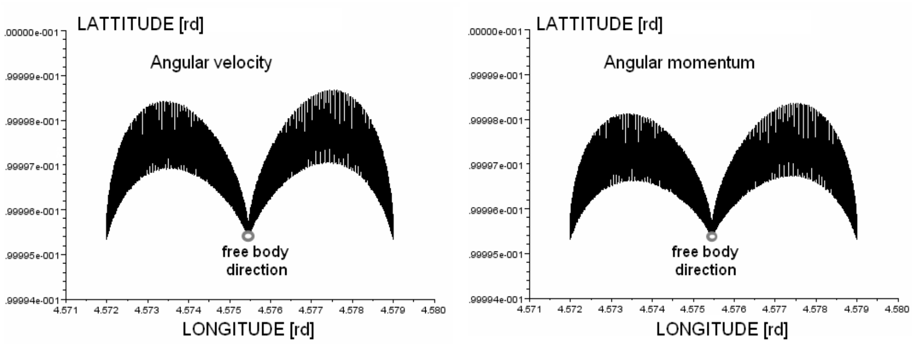

Figure 6. Plot of angular velocity and angular momentum on βλ(latitude and longitude) plane. -free body directions are constant.

The situations change, if SAM assuming is only approximate. Then all changes physical parameters remain periodic, but difference between angular velocities perturbed and free body rise in time (figure 7). This rising is periodical too, but time period is in range of hundreds years and out of interest of this work.

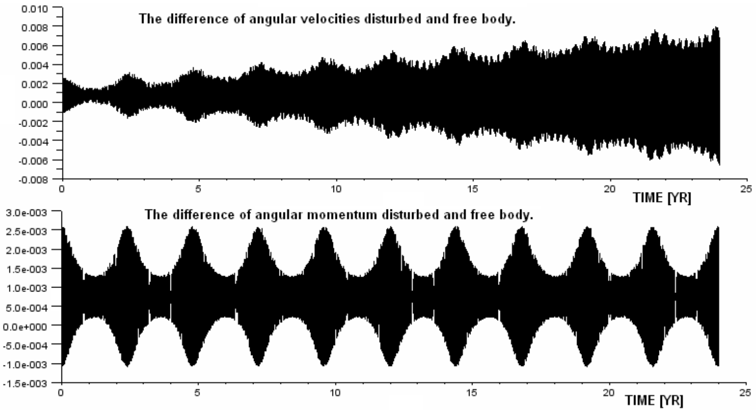

Figure 7. Calculation for Ta=2.4yr, a=0.56, b=0.9, w=[10;20,200].

Precession of a rotation axe is irregular, but still its amplitude in both polar coordinates is limited. A rotation axe of free body exhibits constant obliquity and small libration in longitude. The spin-vector of the free body passes by fixed point in contrast a spin-vector the disturbed one (figure 8). A position of both spin-vectors differs significantly. If an astronomical observations confirm ability of motion different then SAM state, theoretical model with gravitational interaction seems to be more properly.

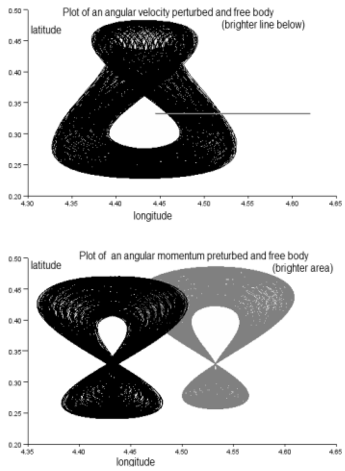

Figure 8. Plot of direction angular velocity and angular momentum in polar coordinates βλ.

All plots of spin-vectors trajectories shows difference of initial and end position after ten orbital revolutions. For NEA asteroids it means about 10-20 years. Some of them are observed since fifty and more years.

The LAM motion is less probably, because this is instability state. For pure LAM motion angular velocity, angular momentum and kinetic energy perturbed body are less then these constant for free body, but still periodic as in SAM motion (figure 9).

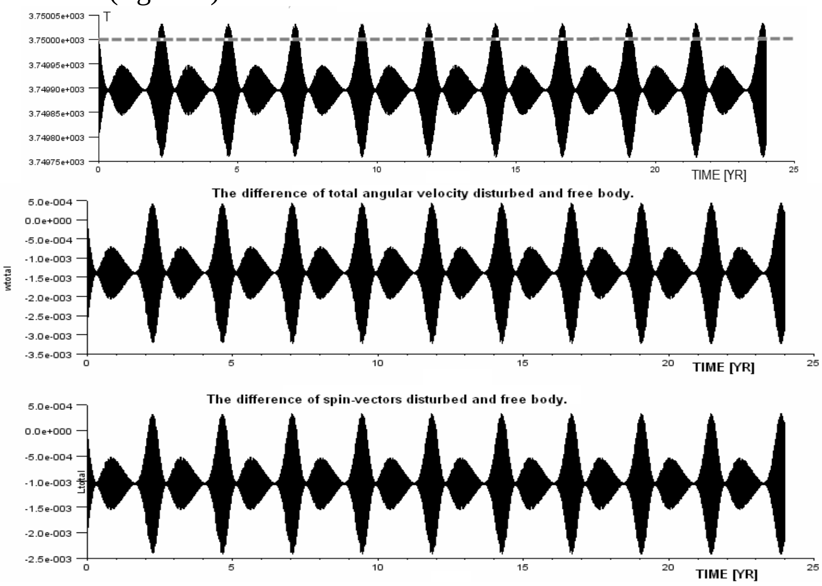

Figure 9. The kinetic energy T of disturbed (black area) and free body (dashed line). The difference of angular velocities and spin-vectors. LAM mode, Ta =2.4yr, wx=200, a=0.75, b=0.9.

The angular velocity disturbed body precesse irregular about x axe (fig. 10), while undisturbed one precesse regularly with constant obliquity.

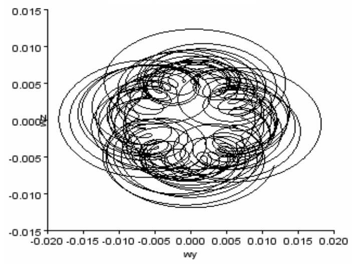

Figure 10. Irregular precession of angular velocity about x axes. LAM mode.

Trajectories of spin-vectors and angular velocities are shown on Figure 11. The obliquities are limited, but longitudes turn perigon many times by each orbital revolution. So by LAM assumption model of rotational motion is very difficult to analysis. All authors make assumption of SAM state of rotational motion, even though experimental data suggest irregular motion NPA type [5,6,10,13,15,16]. This simplifies analysis, but for last discovered NPA rotators may be useless.

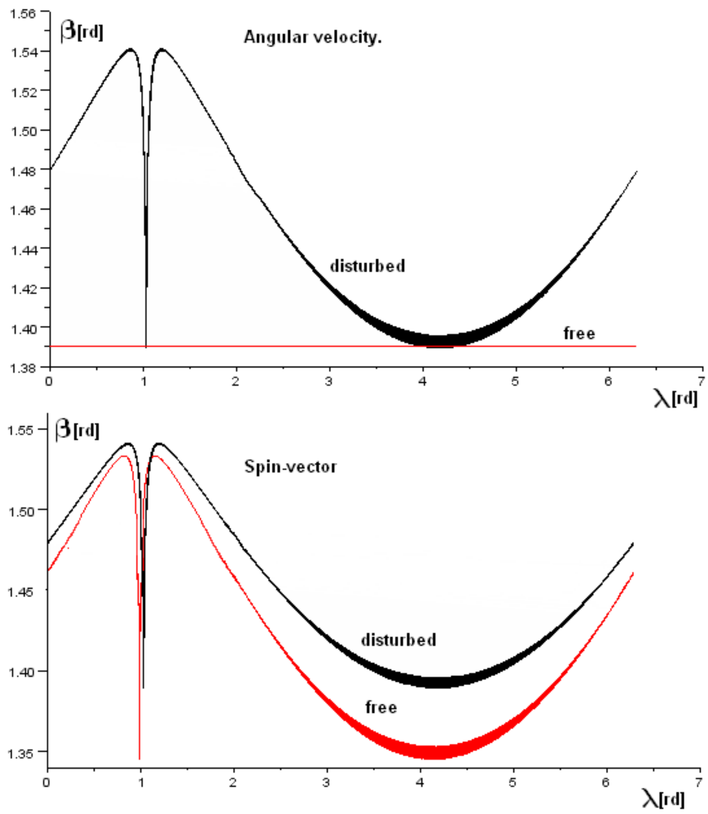

Figure 11. Plot of angular velocity and spin-vector trajectories in polar coordinates. LAM motion like on fig. 9.

NPA rotators called tumblers have in the majority long rotational periods. For analysis experimental data usually is used model of free rotational motion well known as Poinset solution, in absence of external gravitational torque [14]. Rotational motion consists from two motions: rotation bodies about z axis with period T_φ and rotation this axis about spin-vector $\vec{L}$ with period T_ψ, where φ and ψ- Euler angles. In this work computer simulation rotational motion forced by external gravitational torque were made for really existing NPA rotator. For example was chosen planetoid 253Mathilda. The parameters of motion and shape according to paper [6] is: Ta=4.3yr, e=0.265, w_{tot}=90 (calculated from listed rotation period T=417.7hr), a=0.664, b=0.972. Author warns that the uncertainty of period estimation may be wrong by a factor of 2. This means, that T≈200÷800hr.

Assuming SAM state of rotation, we confirm earlier results: small growth angular momentum and velocity and small, regular precession of angular velocity about z axis. But a direction of angular velocity and angular momentum becomes strong unstable and almost chaotic character (figure 12). These components have fundamental meaning in establishing period of asteroid rotation from experimental data.

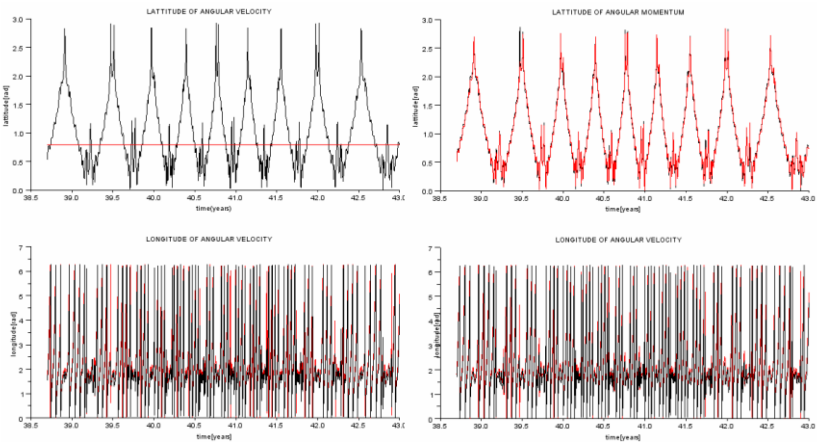

Figure 12. Time variation of angular coordinates angular momentum and angular velocity simulated for 253Mathilda.

Latitude both physical parameters vary from about zero to three radians, longitude turn perigon many times during one orbital period. Trajectories both angular variables in angular coordinates seem chaotic (figure 13,14), but assumption of a SAM state motion, if it is false leads to bad estimation of rotational period.

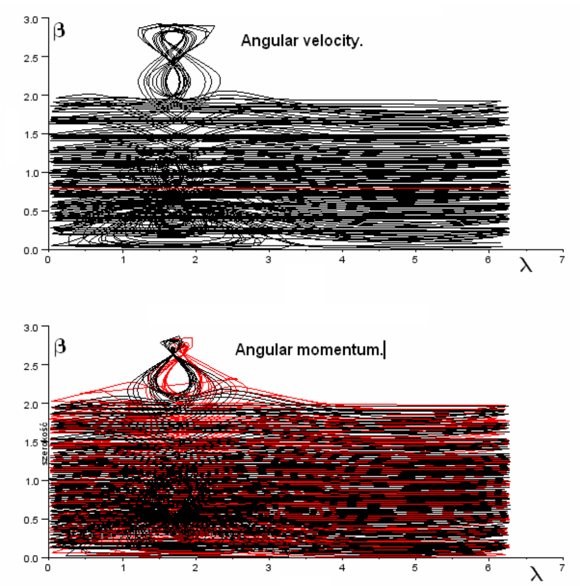

Figure 13. Trajectories plot of angular velocity and angular momentum 253 Mathilda.

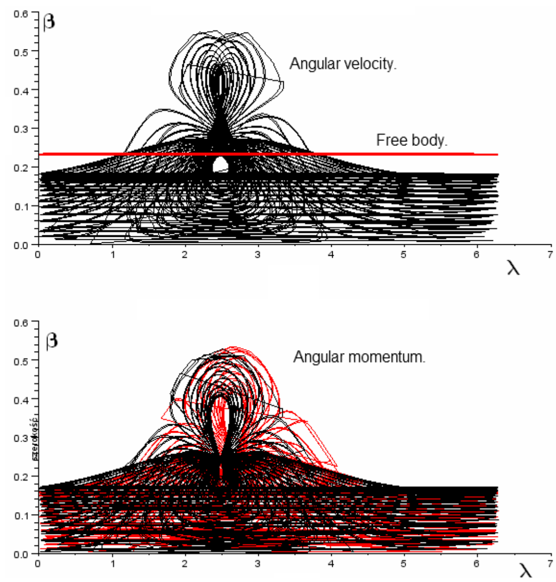

Figure 14. Trajectories plot of angular velocity and angular momentum 253 Mathilda – closer SAM state.

The trajectories presented on fig.14 are computed for rotational motion differs from pure SAM mode only about a few degrees and it may be prevalent in Solar System. The small deviation from rotation about principal axis leads to dramatic changes.

5 Summary.

Presented results shows, that only for rotation about principal short axis Poinset solution of Euler-Liouville equation is reasonable model of rotated asteroids. Photometric lightcurves of asteroids provide information about rotational periods, orientation of axes and shape. If photometric lightcurve exhibit multiperiodical character, longtime deduction needs use some more physical parameters, like gravitational interaction. The gravitational torque is very small compared to kinetical energy, but for tumbling asteroids it is sufficiently to change markedly trajectory of spin-vector in a few year. Its influence is negligible only in pure SAM mode of rotational motion. A direction of angular momentum and angular velocity of asteroids are two important parameters deduced indirectly from astronomical observations. Solution of slow rotated asteroids forced by gravitational torque varies sufficiently fast to change theoretical presumption for time observation range of a few dozen and less years. It seems likely, that good adjusted dynamical Euler-Liouville equation gives better agreement of lightcurves and rotational periods observed for many years asteroids. Uncertainty presented in work [6] parameters confirm the necessity of building more sophisticated models of asteroids rotation, involving all known interaction, like gravitational torque and YORP effect. Gravitational torque really doesn't change marked periods of rotation, kinetical energy or total angular velocity and angular momentum. But its influence for spatial directory of rotational axe and spin-vector direction is significant, so it must be taken into account in future work on this field. Also small change of rotational period the gravitationally forced asteroid must be taken into account in long evolution time prediction.

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